Acta Technica 62, No. 3B/2017, 9–16

Stability analysis of non-linear stochastic dynamic system based on numerical simulation[1]

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Abstract. The stability of nonlinear stochastic dynamical system is a theoretically prominent problem, as it is of great theoretical and practical significance to many projects in reality. Firstly, the author established the basic formula. Secondly, he researches the stability theory of non-linear stochastic dynamical system, and discusses the stability criterion of null solution stability and nonlinear stochastic dynamic system. Finally, he conducts numerical simulation analysis for kinetics response and stability status of engineer structures under seismic conditions. The simulation results provides theoretical basis for the promotion of earthquake resistance of engineer structures.

Key words. Numerical simulation, nonlinear stochastic dynamics simulation, stability.

1. Introduction

At present, the application of nonlinear dynamical system theory in the fields of mathematics, physics, mechanics, and economics has become hot spot in focus. Therefore, it is necessary to get to know the internal mechanism of power system nonlinear phenomena in order to conduct theoretical analysis and numerical analysis. But many problems remain to be solved due to the complicity of non-linear systems. At present, researchers put much emphasis on phenomenon of definite systems. In fact, it is unavoidable that most systems are subject to the interference of stochastic noises, and definite system is idealized system, thus stochastic systems could reveal the natural laws in a more deepgoing and truer way. Many researches prove that noise could determine the evolution of system due to the interactions in between, which probably totally destroy system structures, or engender the system in order instead. Therefore, it is necessary to get profound understanding of the internal

[1]This work is supported by Social Science Research Project of Jiangxi Province of China under Grant No. 17GL33, Science and Technology Research Project of Jiangxi Provincial Department of Education under Grand No. GJJ16113. In addition, the author would particularly like to thank the anonymous reviewers for helpful suggestion.

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mechanism of nonlinear stochastic phenomena and movement patterns, so as to lead it to a favorable direction. It is a research of great scientific significance and practical guidance value.

The existing stochastic systems can be classified to three categories: system with stochastic stimulation, system with stochastic initial conditions, and system with stochastic physical parameters. In reality, there exists many uncertainties in installation, measuring, material and production sectors, which could be described by stochastic variables with some statistical properties, so the number of the third category is the largest. Due to the increasing demand for precision and accuracy of actual model, stochastic system, especially those with stochastic parameters have been applied more widely to describe the dynamic relationship between various objects.

The 20th century has witnessed the booming of stability theories. Lasalle proposed the Lasalle invariance theory based on Lyapunov's stability theory and Birkhoff's invariant set. More than 50 years ago, with the introduction of stochastic integral by Itó, stability theory of stochastic dynamic system has also experienced rapid development, but theoretical results about the stability of nonlinear stochastic dynamic systems are still far from enough, especially those with stochastic parameters. The stochastic dynamical systems with some known statistics information are much closer to actual models, thus their stability study is of great practical and theoretical significance and deserves more attention. The existing methods to deal with stochastic dynamical systems with stochastic parameters include: Monte Carlo method, which is easy and widely adopted but time consuming; stochastic finite element method, which is time saving, but on the premise that the stochastic variable is small; and orthogonal polynomial approximation method based on orthogonal polynomial expansion theory, which is free from the limitations of the previous two methods, and widely applied into the study of evolution stochastic response, bifurcation and chaos of stochastic systems.

2. Basic equation of nonlinear stochastic dynamic system

The dynamics model with multiple freedom degree could be expressed as [1]

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{C}\dot{\boldsymbol{X}} + f(\boldsymbol{X}) = \boldsymbol{F}(t)\,, \tag{1}$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{C}$ means the damping matrix, $f(\cdot)$ indicates the recuperability function, $\ddot{\boldsymbol{X}}$ represents the acceleration, $\dot{\boldsymbol{X}}$ is the speed, $\boldsymbol{X}$ is the displacement, and $\boldsymbol{F}(t)$ is the exciting force random vector.

After setting $\boldsymbol{F}(t) = -\boldsymbol{M}\boldsymbol{I}\ddot{x}_g(t)$, the nonlinear stochastic system model can be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{C}\dot{\boldsymbol{X}} + f(\boldsymbol{X}) = -\boldsymbol{M}\boldsymbol{I}\ddot{x}_g(t)\,, \tag{2}$$

where $\boldsymbol{I} = [1, 1, \cdots, 1]^{\mathrm{T}}$ and $\ddot{x}_g(t)$ represents the acceleration of the excitation force.

Set now $Y = (\dot{\boldsymbol{X}}^{\mathrm{T}}, \boldsymbol{X}^{\mathrm{T}})^{\mathrm{T}}$. Then the state equation of nonlinear stochastic dynamic systems can be converted to the form [2]

$$\dot{\boldsymbol{Y}} = \boldsymbol{A}(\boldsymbol{Y}, t) + \boldsymbol{B}\boldsymbol{F}(t)\,, \tag{3}$$

where

$$\boldsymbol{A}(\boldsymbol{Y}, t) = \begin{bmatrix} -\boldsymbol{M}^{-1}\boldsymbol{C}\dot{\boldsymbol{X}} - \boldsymbol{M}^{-1}f(\boldsymbol{X}) \\ \dot{\boldsymbol{X}} \end{bmatrix}, \; \boldsymbol{B} = \begin{bmatrix} \boldsymbol{M}^{-1} \\ 0 \end{bmatrix}.$$

Set $Y_i = X_{2i-1}$, $f_{2i-1} = X_{2i-1}$, $g_{2i,j} = a_{ij}$, and $g_{2i-1,j} = 0$. Now the mathematical model of the nonlinear stochastic system can be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}t} X_i = f_i(\boldsymbol{X}) + g_{ij}(\boldsymbol{X}) A_j(t)\,, \tag{4}$$

where X_i represents the component of the state vector, $g_{ij}(\boldsymbol{X})$ signifies the Markov process vector, and $p(\boldsymbol{X}, t)$ is the corresponding probability density function. The control equation of $p(\boldsymbol{X}, t)$ can be expressed as

$$\frac{\partial}{\partial x_i}[f_j(\boldsymbol{X}), p(\boldsymbol{X})] - \frac{1}{2}\frac{\partial^2}{\partial x_i \partial x_j}[g_{ij}(\boldsymbol{X}), p(\boldsymbol{X})] = 0\,. \tag{5}$$

The probability density function $p(\boldsymbol{X}, t)$ is subject to the following conditions

$$\lim_{X_i \to \pm\infty} f_j(\boldsymbol{X}) p(\boldsymbol{X}, t) = 0 \tag{6}$$

and

$$\lim_{X_i \to \pm\infty} \frac{\partial[g_{ij}(\boldsymbol{X}) p(\boldsymbol{X}, t)]}{\partial x_i} = 0\,. \tag{7}$$

3. Stability theory of nonlinear stochastic dynamic system

3.1. Null solution stability

Definition 1: Set $q > 0$, $\forall X_0 \in [0, 1]$, X_0 is a random variable, and $X(t, X_0)$ is the solution of equation (5). If there exists negative q moment index, the expected null solution q moment index of equation (5) is stable; on the contrary, null solution q moment index of equation (5) is unstable[3]. Figures 1 and 2 are the corresponding schematic diagrams.

If X_0 is a random number, Fig. 1 shows that if $t \to \infty$, the absolute expectation moment index $E\,|X(t, X_0)|^q$ of $X(t, X_0)$ would decrease to zero. Figure 2 shows that if $t \to \infty$, the absolute expectation moment index $E\,|X(t, X_0)|^q$ of $X(t, X_0)$ would increase to infinity.

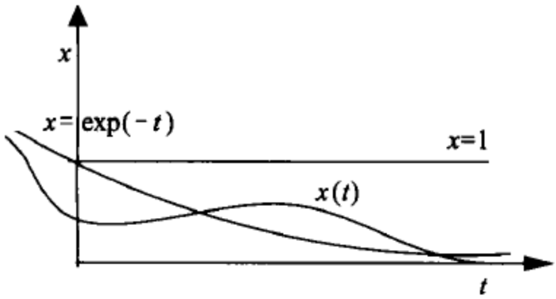

Fig. 1. Schematic diagram about the stability of null solution q moment expectation index

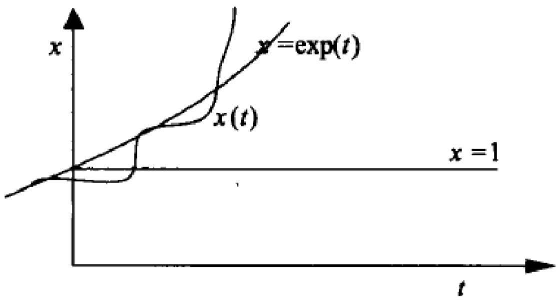

Fig. 2. Schematic diagram about the instability of null solution q moment expectation index

3.2. Stability criterion

Lemma: For nonlinear stochastic dynamic systems [4] there holds

$$\frac{\mathrm{d}X(t, X_0)}{\mathrm{d}t} = F(t, X(t)) + G(t, X(t))B(t),\ X(t_0) = X_0 \,. \tag{8}$$

Solution of the equation (8) is defined as $X(t) = X(t, X_0)$. Assume the following formula is true:

$$d_1 \left|x\right|^q \le V(t, X) \le d_2 \left|x\right|^q \ , \tag{9}$$

where d_1 and d_2 are constants greater than zero. Quantity $V(t, X)$ is a smooth function.

(a) If there exists a normal number λ which is subject to the following formula: $LV(t, X) \le -\lambda V(t, X)$, then the following inequality is true:

$$E\left|x(t, X_0)\right|^q < \frac{c_2}{c_1}\left|x_0\right|^q e^{-\lambda t} \,. \tag{10}$$

(b) If there exists a normal number λ which is subject to the following formula:

$LV(t, X) \geq \lambda V(t, X)$, then the following inequality is true:

$$E\left|x(t, X_0)\right|^q \geq \frac{c_1}{c_2}\left|x_0\right|^q e^{\lambda t}, \tag{11}$$

where

$$LV(t, X) = V_t(t, X) + V_x(t, X)F(t, X) + \frac{1}{2}G^2(t, X)V_{xx}(t, X).$$

4. Simulation analyses about the stability of nonlinear stochastic dynamic system

Consider engineering structures earthquake for instance and let $\boldsymbol{F}(t)$ represent the seismic excitation. The basic parameters of the engineering structure are listed as follows: the composite material layered plate structure is used in this research, the structure is formed by sixth layered fibers, and the size of the engineering structure is 1000 mm×500 mm, and the thickness of the plate is 20 mm. The excitation is stochastic pulse load. The deformation of engineering structures is a point of focus. Programming simulation programs with MATLAB software, the corresponding simulation results are as follows:

4.1. The dynamic response of engineering structures under seismic conditions

Figures 3 and 4 show the dynamics simulation results of engineering structures under seismic excitation. Figure 3 is the response result of engineering structure deformation under seismic excitation, which changes dramatically at initial stage and then stabilizes gradually. Figure 4 is the response result of engineering structure hysteresis deformation under seismic excitation, which changes dramatically at initial stage and then stabilizes gradually.

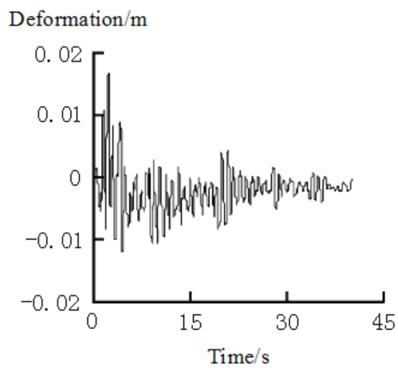

Fig. 3. Response curve of engineering structure deformation

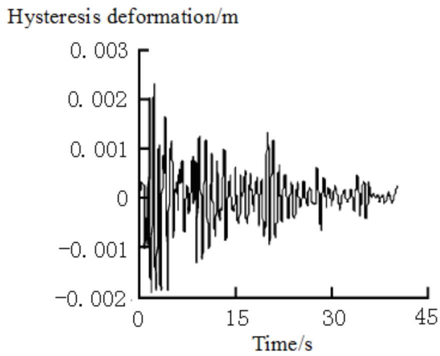

Fig. 4. Response curve of engineering structure hysteresis deformation

4.2. Stability analysis

Stability simulation of nonlinear stochastic dynamic systems is shown in figure 5, from which we could notice that engineering structures are unstable under seismic conditions, and they are stable for long at no time. Therefore, it is required to reinforce the structure, in order to improve the seismic capacity of structures and improve the stability of structures.

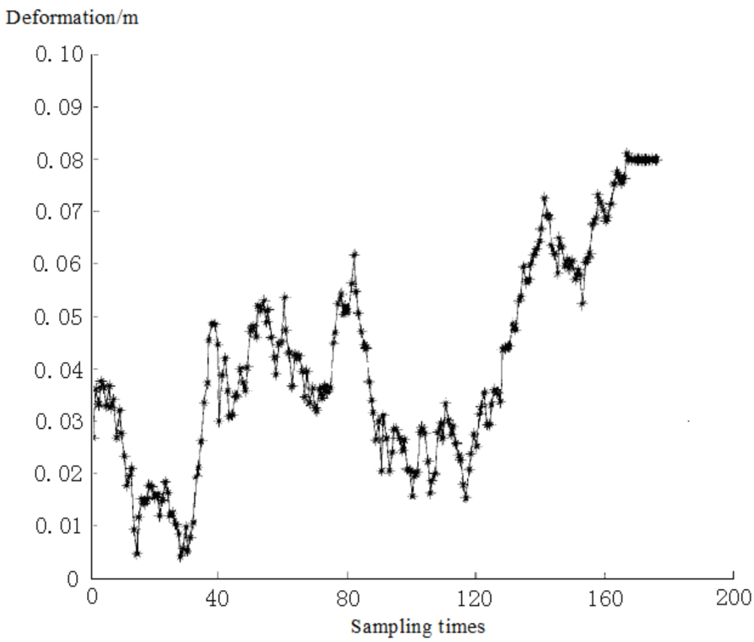

Fig. 5. Simulation curve about the stability of engineering structures

5. Conclusion

In this paper, the author conducts systematic research on the stability of nonlinear stochastic dynamical systems. He analyses the mathematical models of nonlinear stochastic dynamical systems and stability theorem and criterion. Moreover, the author conducts numerical simulation on the engineering structures under seismic conditions. The results reveal the stability degree of structures, which provides valid theoretical basis for the improvement of seismic capacity and ensures the safety and stability of engineering structures. Therefore, it is very promising in practice.

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Received April 30, 2017